Two-scale FEM for the linear Eddy Current Problem in 3D

K. Hollaus, and J. Schöberl

Vienna University of Technology, Wiedner Hauptstr. 8, A-1040 Vienna, Austria
E-mail: karl.hollaus@tuwien.ac.at

***Abstract*—The simulation of eddy current losses in laminated iron cores by the finite element method is of great interest in desinging of electrical machines. Modeling each lamination individually requires many elements and leads to an inappropriate large equation system. To overcome this problem two-scale FEM is proposed to compute the losses efficiently. Two-scale FEM is explained and the accuracy and the computational costs are presented by a representative numerical example.**

I. Introduction

An efficient and accurate simulation of the eddy current losses in laminated cores is still a challenging task [1]-[7]. Brute force methodes apply anisotropic material properties in finite element models ([2]-[3]) yielding losses which are to small because the losses caused by the main magnetic flux parallel to the laminations are neglected. Therefore, the solution obtained by this method is frequently corrected in a second step exploiting different approaches, i.e. [4]-[5]. Homogenization methods, where the main magnetic flux is considered directly, have been proposed in [6] and [7].

Contrary to [4]-[7] the present method is based on multi-scale FEM and accounts for an air gap between the iron sheets. An ansatz of the two-scale FEM has been derived for the magnetic vector potential describing the eddy currents in laminated iron cores and capable to treat a laminated core efficiently as a bulk without the necessity to model the laminations individually. The method requires only a matrix-vector and a vector-vector multiplication to calculate the losses. Only linear material properties are considered. The accuracy and the computational costs of the multi-scale FEM are evaluated by a reference solution of a small numerical example in section III.

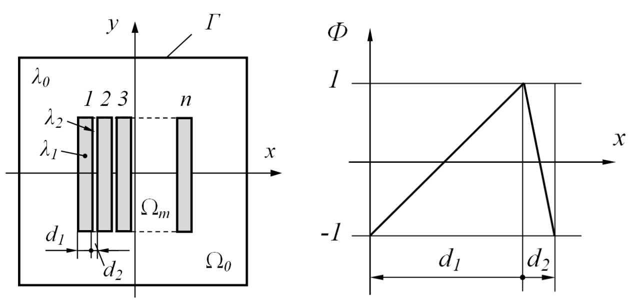

Fig. 1. Boundary value problem with a laminated media, top view (left) and periodic micro-shape function $\phi(x)$ (right).

II. Two-Scale FEM for the Eddy Current Problem

For the sake of simplicity the following case of an eddy current problem is studied to explain the two-scale FEM. The computational domain Ω of the eddy current problem consists of a laminated medium Ω_m surrounded by air Ω_0 (see Fig. 1):

$$\Omega = \Omega_m \cup \Omega_0 \tag{1}$$

The material parameters λ_0, λ_1 and λ_2 are valid in air, in the laminations and in the gap between them, respectively, and stand for the magnetic permeability μ and conductivity σ, respectively. The weak form of the eddy current problem in the time harmonic case

$$\int_\Omega \mu^{-1} \ \operatorname{curl}\boldsymbol{A} \ \operatorname{curl}\boldsymbol{v} \ d\Omega + j\omega \int_{\Omega_m} \sigma \boldsymbol{A}\boldsymbol{v} \ d\Omega = 0 \tag{2}$$

for the magnetic vector potential $\boldsymbol{A}$ with appropriated boundary conditions on Γ of the eddy current problem is considered.

Based on observations the two-scale ansatz

$$\boldsymbol{A} = \boldsymbol{A}_0 + \phi \begin{pmatrix} 0 \\ A_1 \\ A_2 \end{pmatrix} + \nabla(\phi w) \tag{3}$$

has been derived. In (3) $\boldsymbol{A}_0$ means the mean value, ϕ times the vector with the entries A_1 and A_2 models currents parallel to the laminations, i.e. $(\boldsymbol{J})_y$ and $(\boldsymbol{J})_z$ according to Fig. 1, and the last term takes account of the normal component of current density $(\boldsymbol{J})_x$. The quantities A_1, A_2 and w are scalar functions. The behavior of the micro-shape function ϕ in x-direction is sketched in Fig. 1, and it is constant in the y- and z-direction. Introducing (3) into (2) leads to

$$\begin{aligned} &\int_\Omega \mu^{-1} \Big[\operatorname{curl}\left(\boldsymbol{A}_0 + \phi(0, A_1, A_2)^T + \nabla(\phi w)\right) \\ &\qquad \cdot \operatorname{curl}\left(\boldsymbol{v}_0 + \phi(0, v_1, v_2)^T + \nabla(\phi q)\right) \Big] \, d\Omega \\ &+ j\omega \int_\Omega \sigma \Big[\left(\boldsymbol{A}_0 + \phi(0, A_1, A_2)^T + \nabla(\phi w)\right) \\ &\qquad \cdot \left(\boldsymbol{v}_0 + \phi(0, v_1, v_2)^T + \nabla(\phi q)\right) \Big] \, d\Omega = 0, \end{aligned} \tag{4}$$

where the test functions v_1, v_2 and q vanish in Ω_0.

Simple manipulations and neglecting the derivative of A_1 and A_2 the first integral in (4) reads as

$$A(\overline{\boldsymbol{A}}_0, \overline{A}_1, \overline{A}_2; \overline{\boldsymbol{v}}_0, \overline{v}_1, \overline{v}_2) = \int_\Omega \begin{pmatrix} \operatorname{curl}\overline{\boldsymbol{A}}_0 \\ \overline{A}_1 \\ \overline{A}_2 \end{pmatrix}^T \overline{S}_1 \begin{pmatrix} \operatorname{curl}\overline{\boldsymbol{v}}_0 \\ \overline{v}_1 \\ \overline{v}_2 \end{pmatrix} d\Omega, \tag{5}$$

where

$$\overline{S}_1 = \begin{pmatrix} \overline{\nu} & 0 & 0 & 0 & 0 \\ 0 & \overline{\nu} & 0 & 0 & -\overline{\nu\phi_x} \\ 0 & 0 & \overline{\nu} & \overline{\nu\phi_x} & 0 \\ 0 & 0 & \overline{\nu\phi_x} & \overline{\nu\phi_x^2} & 0 \\ 0 & -\overline{\nu\phi_x} & 0 & 0 & \overline{\nu\phi_x^2} \end{pmatrix}$$

with the magnetic reluctivity $\nu = \mu^{-1}$ and $\phi_x = \frac{\partial \phi}{\partial x}$. Note that $\frac{\partial \phi}{\partial y} = 0$ and $\frac{\partial \phi}{\partial z} = 0$. The coefficients in $\overline{S}_1$ were averaged across the laminations [8]. Averaged coefficients are indicated by the bar. The second integral in (4) yields

$$B(\overline{\boldsymbol{A}}_0, \overline{A}_1, \overline{A}_2, \overline{w}; \overline{\boldsymbol{v}}_0, \overline{v}_1, \overline{v}_2, \overline{q}) = j\omega \int_\Omega \overline{\boldsymbol{A}}^T \; \overline{M_1} \; \overline{\boldsymbol{v}} \, d\Omega, \quad (6)$$

where $\overline{M}_1$ is a sparse and symmetric 9 by 9 matrix with the entries

$$a = \overline{\sigma}, \; b = \overline{\sigma \phi^2}, \; c = \overline{\sigma \phi_x^2} \; \text{ and } \; d = \overline{\sigma \phi_x}, \; \text{respectively,}$$

$$\overline{\boldsymbol{A}} = \left((\overline{\boldsymbol{A}}_0)_x, (\overline{\boldsymbol{A}}_0)_y, (\overline{\boldsymbol{A}}_0)_z, \overline{A}_1, \overline{A}_2, \overline{w}, \partial_x \overline{w}, \partial_y \overline{w}, \partial_z \overline{w} \right)^T$$

and

$$\overline{\boldsymbol{v}} = \left((\overline{\boldsymbol{v}}_0)_x, (\overline{\boldsymbol{v}}_0)_y, (\overline{\boldsymbol{v}}_0)_z, \overline{v}_1, \overline{v}_2, \overline{q}, \partial_x \overline{q}, \partial_y \overline{q}, \partial_z \overline{q} \right)^T.$$

Considering (5) and (6) the problem formulation for the two-scale finite element method reads as follows: Find

$(\overline{\boldsymbol{A}}_{0h}, \overline{A}_{1h}, \overline{A}_{2h}, \overline{w}_h) \in V := \{(\overline{\boldsymbol{A}}_{0h}, \overline{A}_{1h}, \overline{A}_{2h}, \overline{w}_h) : \overline{\boldsymbol{A}}_{0h} \in \mathcal{U}_h, \; \overline{A}_{1h}, \; \overline{A}_{2h} \in \mathcal{V}_h, \; \overline{w}_h \in \mathcal{W}_h$ and $\overline{\boldsymbol{A}}_{0h} \times \boldsymbol{n} = \alpha_h$ on $\Gamma\}$,

such that

$$A(\overline{\boldsymbol{A}}_{0h}, \overline{A}_{1h}, \overline{A}_{2h}; \overline{\boldsymbol{v}}_{0h}, \overline{v}_{1h}, \overline{v}_{2h}) + B(\overline{\boldsymbol{A}}_{0h}, \overline{A}_{1h}, \overline{A}_{2h}, \overline{w}_h; \overline{\boldsymbol{v}}_{0h}, \overline{v}_{1h}, \overline{v}_{2h}, \overline{q}_h) = 0$$

for all

$(\overline{\boldsymbol{v}}_{0h}, \overline{v}_{1h}, \overline{v}_{2h}, \overline{q}_h) \in V_0 := \{(\overline{\boldsymbol{v}}_{0h}, \overline{v}_{1h}, \overline{v}_{2h}, \overline{q}_h) : \overline{\boldsymbol{v}}_{0h} \in \mathcal{U}_h, \overline{v}_{1h}, \; \overline{v}_{2h} \in \mathcal{V}_h, \; \overline{q}_h \in \mathcal{W}_h$ and $\overline{\boldsymbol{v}}_{0h} \times \boldsymbol{n} = \boldsymbol{0}$ on $\Gamma\}$,

where $\mathcal{U}_h$ is a finite element subspace of $H(\text{curl}, \Omega)$, $\mathcal{V}_h$ a finite element subspace of $L_2(\Omega_m)$ and $\mathcal{W}_h$ a finite element subspace of $H^1(\Omega_m)$, respectively. The micro-shape function ϕ is in the space of periodic and continuous functions $H_{per}(\Omega_m)$. The index h stands for finite element discretization.

III. Numerical Example

The numerical problem consists of a cubic domain Ω_0 with an edge length of 10mm. A laminated iron cube Ω_m with an edge length of 5mm and n laminations is arranged symmetrically in the center of Ω_0. Boundary conditions are prescribed for the tangential component of the magnetic vector potential $(\mathbf{A_0})_\mathbf{t}$ on Γ such that a homogenous magnetic flux density of $1.0 Vs/m^2$ would be obtained without the iron cube. A rather unfavorable fill factor of $f = 0.95$, a relative permeability of $\mu_r = 1000$, a conductivity of $\sigma = 2 \cdot 10^6 S/m$ and a frequency of $50Hz$ were selected. This leads to a penetration depth of

$$\delta = \sqrt{\frac{2}{2\pi f \mu \sigma}} = 1.6mm$$

in iron. The gap between the laminations and Ω_0 was assumed to be air.

To study the accuracy of the two-scale FEM the eddy current losses obtained by the two-scale FEM are compared with those obtained by the reference model in which the laminations are modeled individually.

The results are summarized in Table I. A fairly good agreement can be observed. The error increases with the thickness $d = d_1 + d_2$ (see Fig. 1) as expect.

TABLE I
COMPARISON OF EDDY CURRENT LOSSES

Losses in mW		
n	Reference solution	Two-scale FEM
10	2.044	1.76
20	0.642	0.671
40	0.286	0.298

n No. laminations

The corresponding model with anisotropic conductivity [2] and anisotropic magnetic reluctivity [3] yields losses of 0.106mW, the two-scale FEM model 0.105mW assuming 100 laminations.

To figure out the computational costs the number of degrees of freedom (NDOF) are given in Table II for the reference model. The two-scale FEM model required 236 002 unknowns for all simulaions. It can easily be seen that the memory requirement of the the two-scale model is essentially smaller than that of the reference models. Consequently, the two-scale FEM solutions could be calculated by far more faster than the reference solutions.

TABLE II
COMPUTATIONAL COSTS

Reference solution	
n	NDOF
10	294 840
20	2 010 288
40	4 615 815

n No. laminations, NDOF No. degrees of freedom

References

[1] S. Nogawa, M. Kuwata, T. Nakau, D. Miyagi, and N. Takahashi, "Study of Modeling Method of Lamination of Reactor Core," IEEE Trans. Magn., vol. 42, no. 4, pp. 1455-1458, April 2006.

[2] V. C. Silva, G. Meunier, and A. Foggia, "A 3D finite element computation of eddy currents and losses in laminated iron cores allowing for the electric and magnetic anisotropy," IEEE Trans. Magn., vol. 31, pp. 2139-2141, May 1995.

[3] H. Kaimori, A. Kameari, and K. Fujiwara, "FEM Computation of Magnetic Field and Iron Loss in Laminated Iron Core Using Homogenization Method," IEEE Trans. Magn., vol. 43, no. 4, pp. 1405-1408, April 2007.

[4] K. Hollaus, and O. Bíró, "Estimation of 3D eddy currents in conducting laminations by an anisotropic conductivity and a 1Danalytical solution," COMPEL, vol. 18, no. 3, pp. 494-503, 1999.

[5] K. Hollaus, and O. Bíró, "A FEM formulation to treat 3D eddy current in laminations," IEEE Trans. Magn., vol. 36, no. 5, pp. 1289-1292, Sep. 2000.

[6] P. Dular et al., "A 3-D magnetic vector potential formulation taking eddy currents in lamination stacks into account," IEEE Trans. Magn., vol. 39, pp. 1147-1150, May 2003.

[7] L. Krähenbühl, P. Dular, T. Zeidan, and F. Buret, "Homogenization of Lamination Stacks in Linear Magnetodynamics," IEEE Trans. Magn., vol. 40, no. 2, pp. 912-915, March 2004.

[8] K. Hollaus, and J. Schöberl, "Homogenization of the Eddy Current Problem in 2D," presented at 14th International IGTE Symposium, Graz, Austria, Sept. 2010.